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AN ANALYSIS OF A COUNTERFLOW SPRAY COOLING TOWER

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Abstract—The cooling tower analyzed in this study is void of fill. It is vertical with the air stream moving uniformly upwards and the water stream, dispersed into droplets, moving uniformly down. The droplets are introduced at the top of the tower with zero velocity, uniform temperature and a known size distribution. The analysis takes into account the fact that at any given height all droplets are not at the same temperature. Results are presented in the form of a correction factor on a simplified solution which neglects this fact. The effect on the correction factor of all pertinent dimensionless groups is examined.

NOMENCLATURE

a, surface area of droplets contained in unit volume of tower [m^{-1}];

b, slope of saturation line (equation (11)) [J/kg K];

C, specific heat at constant pressure [J/kg K] (C_L, of water, C_G of gas);

C_D, drag coefficient on droplet [dimensionless];

F_T, correction factor defined by equation (39) [dimensionless];

$f_n(x, z)$, $f_n(x, z)\,\partial x$ is the fraction of the total number of droplets at height z which have diameter between x and $x+\partial x$ [m^{-1}];

$f_{no}(x)$, $f_{no}(x)\,\partial x$ is the fraction of the total number of droplets introduced at top of tower which have diameter between x and $x+\partial x$ [m^{-1}]; $f_{no}(x) = f_n(x, 0)$;

$f_{vo}(x)$, $f_{vo}(x)\,\partial x$ is the fraction of the total volume of droplets introduced at the top of the tower in a given time interval, having diameter between x and $x+\partial x$

[m^{-1}], $f_{vo} = x^3 f_{no}(x) \Big/ \int_0^{x_u} x^3 f_{no}(x)\,\mathrm{d}x$;

$\mathscr{F}(\xi)$, $\mathscr{F}(\xi)\,\mathrm{d}\xi$ is the fraction of the total volume of droplets introduced at top of tower having dimensionless diameter between ξ and $\xi+\mathrm{d}\xi$, dimensionless;

g, acceleration of gravity [m/s^2];

$H_a(z)$, specific enthalpy of air at height z [J/kg], $H_{ao} = H_a(o)$;

$H_s(t)$, specific enthalpy of air at equilibrium with water at temperature t [J/kg], $H_{so} = H_s(t_o)$;

$H_s(x, z)$, specific enthalpy of air at equilibrium with water droplets at height z and of diameter x [J/kg];

$h_c(x, z)$, convective heat-transfer coefficient from air to droplet of diameter x and at height z [W/m^2 K];

k, thermal conductivity of air [W/m K];

L, a characteristic length, equal to $\left[\frac{\nu^2 \rho_G}{g \rho_L}\right]^{1/3}$ [m];

m, mass flow rate [kg/s], (m_L, of water; m_G, of air);

$N(z)$, total number of droplets per unit volume at height z [m^{-3}];

$NTU(Z)$, number of transfer units of tower of height Z, dimensionless; $NTU_o(Z) = NTU$ corresponding to simple solution—equation (34);

$Nu(Re, We)$, Nusselt number corresponding to convective heat transfer over droplet, at Reynolds number Re and Weber number We [dimensionless]; $= \frac{h_c x}{k}$;

$\hat{N}u(\xi, \mathrm{H})$, average Nusselt number of droplet of dimensionless diameter ξ; for a tower of dimensionless height H [dimensionless], equation (31);

Pr, Prandtl number (ν/α) [dimensionless];

q, local total heat transfer from droplets to air per unit volume of tower [W/m^3];

R, $\frac{m_L C_L}{m_G b}$ [dimensionless];

Re, Reynolds number, $\left(=\frac{V_r x}{\nu}\right)$ [dimensionless];
$T(x, Z)$, residence time of droplet of diameter x in tower of height Z [s];
$t(x, z)$, temperature of droplet of diameter x and at height z [K];
t_o, feed temperature of water to tower [K];
$U(\xi, \eta)$, Reynolds number of droplet of dimensionless diameter ξ and at dimensionless height η [dimensionless], $U(\xi, \eta) = V_r x/\nu$;
$\hat{U}(\xi, \eta)$, average Reynolds number defined by equation (30) [dimensionless];
$V(x, z)$, absolute velocity of droplet of diameter x and at height z [m/s];
V_t, $V_t(x)$, terminal absolute velocity of droplet of diameter x [m/s];
$\hat{V}$, $\hat{V}(x, Z)$, average absolute velocity of droplet of diameter x in tower of height z [m/s] $(= Z/T)$;
V_r, $V_r(x, z)$, relative velocity between droplet and air stream [m/s]; $(V_r = V + V_a)$;
V_{rt}, $V_{rt}(x)$, terminal relative velocity between droplet and air stream [m/s];
$\hat{V}_r$, $\hat{V}_r(x, Z)$, average relative velocity of droplet in tower of height Z [m/s];
V_a, velocity of air rising up tower [m/s];
We, Weber number $\left(\frac{\rho_G V_r^2 x}{\sigma}\right)$ [dimensionless];
x, droplet equivalent spherical diameter [m], $\left(x = \left(\frac{6}{\pi}\,.\,\text{volume of drop}\right)^{1/3}\right)$;
x_{31}, volume–diameter mean diameter of droplets [m]:

$$\frac{1}{x_{31}^2} = \int_o^{x_u} \frac{1}{x^2} f_{ov}(x)\,\mathrm{d}x;$$

x_u, upper limit diameter of droplet size distribution [m];
Z, height of tower [m];
z, distance down from top of tower [m].

Greek letters

α, thermal diffusivity of air [m^2/s];
β, $6\,b/PrC_L$ [dimensionless];
γ, characteristic of spray droplet size distribution defined by equation (17) [dimensionless];
δ, characteristic of droplet size spray distribution defined by equation (17) [dimensionless];
H, $\frac{Z\rho_G}{L\rho_L}$ [dimensionless];
η, $\frac{z\rho_G}{L\rho_L}$ [dimensionless];
θ, $(H_{so} - H_s)/(H_{so} - H_{ao})$ [dimensionless];
κ, $\frac{\rho_G \nu^2}{\sigma L}$ [dimensionless];
ν, kinematic viscosity of air [m^2/s];
ξ, ξ_u, ξ_{31}, ξ_m, $\xi = x/L$ [dimensionless],

$$\xi_u = x_u/L;\ \xi_{31} = x_{31}/L,\ \xi_m = \frac{\xi_u}{1+\gamma};$$

ρ, density [kg/m^3]; ρ_G, of air; ρ_L, of water;
σ, surface tension of water [N/m];
τ, $T\,.\,\frac{\rho_G}{\rho_L}\,.\,\frac{\nu}{L^2}$ [dimensionless];
υ, $\frac{V_a L}{\nu}$ [dimensionless];
ω, $\frac{U_t - \nu\xi}{U_t + \nu\xi} = \frac{V_{rt} - V_a}{V_{rt} + V_a}$.

INTRODUCTION

THE COOLING tower remains the engineer's chief device for dissipating large quantities of heat to the environment. Despite its popularity, the basic theory of this equipment remains essentially unchanged from that first developed by Merkel in 1925. As opposed to other types of heat-transfer equipment, such as heat exchangers, the relevant coefficients are developed purely by empirical methods and have not been examined analytically. Moreover, some of Merkel's basic assumptions are yet to be examined in depth.

The present paper presents an analysis of a counterflow spray cooling tower. This study was prompted by the need to design towers for applications in which, due to salt deposition on the packing and subsequent air-flow blockage, the use of a tower packing is not practical. It is felt that the study may also serve to point out ways of gaining a better understanding and perhaps subsequent improvement of more conventional packed towers. The general approach of this study should also be applicable to spray ponds and canals.

Studies of spray cooling towers have been reported sporadically over the years: Niederman *et al.* [1], Lowe and Christie [2], and Dutkiewics [3] have described experimental investigations while Nottage and Boelter [4] have reported an analytical approach, based on (among other things) droplets of uniform size. The experimental studies show the volumetric coefficient and HTU to depend upon factors, of which, for packed towers, they are traditionally assumed to be independent. The most important of these factors is the tower height, the volumetric coefficient having been found to decrease dramatically as the tower height is increased. This is particularly to be noted in the studies of Lowe and Christie. In some cases this effect may be explained as due to a greater concentration of droplets near the top of the tower, associated with a lower droplet velocity near the top, but this cannot explain all of the results. An explanation suggested by Lowe and Christie is agglomeration of the water droplets; however the very large void fractions which exists in the towers appears to argue against this. An alternate explanation is associated with the fact that the droplets produced by a spray are not of uniform size. A feature of the present analysis is that it does not assume all droplets to be of the same diameter but rather that they have some (known) size distribution.

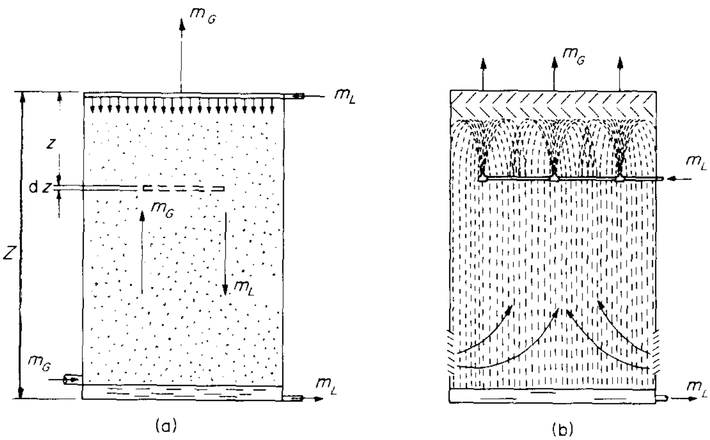

FIG. 1. (a) Model of tower analyzed; (b) Possible tower treated by model.

The spray tower model analyzed in the present study is sketched in Fig. 1(a): droplets fall vertically downward from the top of the tower where they are uniformly distributed, with some known droplet-size distribution and with zero initial velocity; at the same time an air stream rises, uniformly and vertically and in direct counterflow to the droplets. This model may be applied to the type of tower, sketched in Fig. 1(b), where the water is introduced near the top with a set of spray nozzles; it may also have use in modeling the void region between successive slats in a wood packed cooling tower.

ASSUMPTIONS

The assumptions of the present model, in addition to those mentioned above are given in the analysis. However, some of the most important ones are discussed in what follows. The first of these is the assumption that the void fraction (i.e. local volume fraction occupied by gas) of the tower is essentially unity throughout. As a consequence the air velocity (ignoring density changes of the air in passing through the tower) is assumed to be constant throughout the tower and virtually unaffected by the presence of the drops; as a second consequence, the probability of collision or agglomeration of drops is assumed negligible. The relative velocity between air and drop is assumed to be everywhere below that to cause shattering and since neither shattering nor collision nor agglomeration occur, each droplet is assumed to maintain its identity as it passes through the tower. In addition, the alteration of the diameter of each droplet, due to evaporation, is assumed negligible, and therefore each drop is assumed to maintain its diameter over the full height of the tower. The number of droplets having a given diameter entering a control volume is therefore taken equal to the number of that diameter leaving. Each droplet is assumed to be uniform in temperature throughout (although all droplets at a given height are not assumed to be at the same temperature) so that the main resistance to energy transfer is assumed to lie on the air side of the air–water interface and Merkel's equation (with its inherent assumption that the Lewis number is unity) is assumed to apply.

For the purposes of analysis the equilibrium line (i.e. the relation between water temperature and the enthalpy of air in equilibrium with water at that temperature) is assumed linear, the curvature of this line not being expected to have any special pronounced effects on spray tower performance. The results of the analysis are presented in terms of NTU (number of transfer units) and if one evaluates the NTU in the usual way but without assuming a linear equilibrium line, and identifies that NTU with the NTU of the present study, little error should ensue.

Due to water mass continuity considerations, it is clear that a condition of zero droplet velocity at the top of the tower ($z = 0$) is, in fact, impossible if a non-zero water mass flux m_L is to be maintained. What is in fact assumed is that the droplet velocity at $z = 0$ is sufficiently small so as not to have appreciable effect on the residence time of the droplet in the tower, while at the same time sufficiently large that the void fraction is still very close to unity, even at $z = 0$ in line with the earlier assumption relating to void fraction. Due to the large ratio of the densities of water and air, there is in fact a wide range of droplet velocities where both of these conditions are satisfactorily met.

Because the void fraction is essentially unity the effect of interaction between adjacent droplets (of the sort discussed by Yaron and Gal-or [5]) on drop and heat and mass transfer can be ignored and each droplet can be considered to be in an infinite fluid for the purposes of evaluation of their coefficients. The effect of drop acceleration of these coefficients is also ignored, so that steady correlations are applicable.

The spectrum of droplet sizes in practical sprays will include a range of small droplets whose terminal velocities are less than the air velocity and hence travel upward and are removed by the drift eliminators. [To ensure that these droplets do not comprise a significant fraction of the total liquid volume flow, it is necessary to use a spray producing a rather large (of the order of 1 mm dia) median drop size.] For the purposes of the analysis model these small droplets are assumed to comprise a negligible part of the total liquid flow rate and are therefore ignored.

ANALYSIS

Hydrodynamic

Prior to a thermal analysis, a hydrodynamic analysis is required to establish the local velocity of any drop size, $V(x, z)$, the local number size distribution (normalized), $f_n(x, z)$, and local total number density of drops $N(z)$ at any point in the tower.

The values of $V(x, z)$, $f_n(x, z)$ and $N(z)$ at $z = 0$ are denoted by V_o, $f_{no}(x)$ and N_o respectively. $f_{no}(x)$ is assumed to be a known function; V_o will later be taken as zero. A mass balance on droplets entering at $z = 0$ gives:

$$m_L = N_o V_o \int_o^{x_m} \frac{\rho_L \cdot \pi x^3}{6} \cdot f_{no}(x)\,\mathrm{d}x. \tag{1}$$

Equation (1) establishes the value of N_o in terms of known quantities. Referring to Fig. 1, conservation of the mass of droplets having diameter between x and $x + \partial x$, flowing into and out of control volume of height ∂z and unit cross-section gives:

$$f_n(x, z) \,.\, N(z) \,.\, V(x, z) = C(x)$$

where $C(x)$ is a constant with respect to z. To evaluate $C(x)$ we apply the boundary conditions: at $z = 0$, $V(x, z) = V_o$, $f_n(x, z) = f_{no}(x)$ and $N(z) = N_o$. Using equation (1) to evaluate N_o there results:

$$f_n(x, z) \,.\, N(z) = \frac{6}{\pi} \frac{m_L}{\rho_L x^3} \frac{f_{vo}(x)}{V(x, z)}. \tag{2}$$

Determination of the function $V(x, z)$ requires knowledge of the drag law on the droplet, and that law used in this study is due to Hughes and Gilliland [6]. Details of these laws are given in Appendix A; suffice to say that the drag coefficient for a droplet in the Reynolds number range of interest can be expressed as a function of the Reynolds number and the Weber number:

$$C_D = C_D(Re, We). \tag{3}$$

A force balance on a droplet gives:

$$\frac{\partial V}{\partial z} = \frac{1}{V}\left[g - \frac{3}{4}\frac{\rho_G}{\rho_L}\frac{V_r^2}{x} \cdot C_D\left(\frac{V_r x}{\nu}, \frac{\rho_G V_r^2 x}{\sigma}\right)\right]. \tag{4}$$

Solution of this differential equation, with the appropriate boundary condition: $V(x, o) = 0$ gives the required velocity distribution $V(x, z)$, and, through equation (2), the droplet concentration $f_n(x, z) \,.\, N(z)$.

Thermal analysis

Consider a volume element of depth ∂z and unit cross-sectional area. The surface area of droplets with diameter between x and $x + \partial x$ and contained in this volume element is:

$$\partial^2 a = \pi x^2 f_n(x, z) N(z) \partial x\, \partial z. \tag{5}$$

Assuming Merkel's approximation, the total heat transferred (sensible plus latent) from these droplets is:

$$\partial^2 q = \frac{h_c(x, z)}{C_G}\left[H_s\big(t(x, z)\big) - H_a(z)\right] \partial^2 a. \tag{6}$$

An energy balance on the droplets entering and leaving this volume element, yields:

$$\left[C_L \cdot \rho_L \cdot \frac{\pi x^3}{6} \cdot f_n(x, z) \,.\, N(z) \,.\, V(x, z)\right] \cdot \frac{\partial t(x, z)}{\partial z} = \frac{\partial^2 q}{\partial z\, \partial x}. \tag{7}$$

An energy balance on the air stream entering and leaving the volume element yields:

$$m_G \frac{\mathrm{d}H_a(z)}{\mathrm{d}z} = -\int_o^{x_u} \frac{\partial^2 q}{\partial x\, \partial z} \partial x. \tag{8}$$

Combining equations (5)–(8) and, in addition, equation (2) yields the following pair of coupled integro-differential equations:

$$\frac{\partial t(x,z)}{\partial z} = -\frac{6h_c(x,z)\big(H_s(t(x,z)) - H_a(z)\big)}{\rho_L C_L C_G x V(x,z)} \tag{9}$$

$$\frac{\mathrm{d}H_a(z)}{\mathrm{d}z} = \frac{C_L m_L}{m_G}\int_o^{x_u}\frac{\partial t(x,z)}{\partial z} f_{v,o}(x)\,\partial x \tag{10}$$

and this pair together with the equilibrium relation, $H_s = H_s(t)$ fully describe the thermal behaviour of the model tower. As has already been mentioned, the present analysis assumes that the equilibrium line is linear so that:

$$\frac{\mathrm{d}H_s(t)}{\mathrm{d}t} = b \tag{11}$$

where "b" is a constant. Introduction of equation (11) into equation pair (9), (10) yields

$$\frac{\partial H_s(x,z)}{\partial z} = -\frac{6bh_c(x,z)\big(H_s(x,z) - H_a(z)\big)}{\rho_L C_L G_G x V(x,z)} \tag{12}$$

$$\frac{\mathrm{d}H_a(z)}{\mathrm{d}z} = \frac{m_L C_L}{m_G b}\int_o^{x_u}\frac{\partial H_s(x,z)}{\partial z}\cdot f_{v,o}(x)\,\partial x \tag{13}$$

where $H_s(x,z) = H_s\big(t(x,z)\big)$.

An appropriate set of boundary conditions for these equations may be obtained by specifying the inlet air and water states, i.e. $H_s(x,o) = H_{so} = H_s(t_o)$ and $H_a(Z) = H_{a1}$ where Z is the tower height. An equally acceptable, but more workable set is obtained by specifying both the air and water states at the top of the tower:

$$H_s(x,0) = H_{so};\; H_a(0) = H_{ao}. \tag{14}$$

This set permits a finite difference procedure for solving the equations to proceed down from the top of the tower. The resultant air enthalpy at any height z corresponds to a solution for a tower of that height and with that inlet air enthalpy. (When the equations are de-dimensionalized, this particular solution is found to correspond to a range of actual conditions. In this manner the full range of tower heights is solved for by one "march" down the tower.)

Equation (13), with boundary conditions (14) can be integrated directly with respect to z to give:

$$H_a(z) = H_{ao} - \frac{m_L C_L}{m_G b}\left[H_{so} - \int_o^{x_u} H_s(x,z) f_{v,o}(x)\,\partial x\right]. \tag{15}$$

This expression for $H_a(z)$ may be substituted into equation (12) so as to reduce equation pair (12) and (13) to one integro-differential equation for $H_s(x,z)$. The full equation will not be written here for brevity but rather written in dimensionless form in a later section.

In order to solve equation pair (12) and (15) it is clearly necessary to evaluate the convective coefficient $h_c(x,z)$, and to specify the feed droplet size distribution function $f_{ov}(x)$. For the former, the correlation due to Ranz and Marshall [7], with a minor modification to correct for droplet distortion, has been used; the details are given in Appendix B. Accordingly, the Nusselt number for forced convection over a droplet will be expressed as a function (assumed known) of the Reynolds number and Weber number:

$$Nu = Nu(Re, We). \tag{16}$$

From the several droplet size distribution functions which have been put forward, the special upper limit function due to Mugele and Evans [8] has been chosen for the present study since it has been demonstrated by these authors as being superior in many respects, and since it has been found to fit accurately spray data for nozzles used for cooling towers. The distribution is given by:

$$f_{o,v}(x) = \frac{x_u\delta}{x(x_u - x)\sqrt{\pi}}\exp\left[-\left(\delta\ln\frac{\gamma x}{x_u - x}\right)^2\right] \tag{17}$$

where x_u is the maximum droplet size found in the spray and γ and δ are dimensionless constants.

Once $H_s(x,z)$ has been determined by solving equations (12) and (15), the number of transfer units of the tower of any height z can be evaluated. For the purposes of a spray tower, the NTU will be defined by:

$$NTU(Z) = \int_{H_a(Z)}^{H_{ao}}\frac{\mathrm{d}H_a(z)}{\bar{H}_s(z) - H_a(z)} \tag{18}$$

where $\bar{H}_s(z)$ is the bulk mean "equilibrium air enthalpy" of the droplets at height z:

$$\bar{H}_s(z) = \int_o^{x_u} f_{o,v}(x) H_s(x,z)\,\mathrm{d}x. \tag{19}$$

By combining with equation (15), equation (18) can be integrated directly to obtain the required expression for NTU:

$$NTU(Z) = \frac{R}{R-1}\ln\left[\frac{RH_{so} - H_{ao} - (R-1)\bar{H}_s(Z)}{H_{so} - H_{ao}}\right]. \tag{20}$$

DIMENSIONLESS GOVERNING EQUATIONS

The equation obtained by eliminating $H_a(z)$ between (12) and (15), together with equation (4), the appropriate boundary conditions and the subsidiary equations for C_D, Nu and $f_{o,v}$ constitute the governing equations for the tower. Written in dimensionless form they reduce to:

$$\frac{\partial \theta}{\partial \eta}=\left(\frac{\beta Nu(U,\kappa U^2/\xi)}{\xi(U-\upsilon\xi)}\right).\left[1-\theta+R\int_o^{\xi_u}\mathscr{F}(\xi)\theta\partial\xi\right] \quad (21)$$

$$\frac{\partial U}{\partial \eta}=\frac{1}{(U-\upsilon\xi)}\left[\xi^2-\tfrac{3}{4}.\frac{C_D\left(U,\kappa\frac{U^2}{\xi}\right)U^2}{\xi}\right] \quad (22)$$

with boundary conditions:

$$\theta(\xi,0)=0;\quad U(\xi,0)=\upsilon\xi. \quad (23)$$

$\mathscr{F}(\xi)$ is the dimensionless volumetric droplet size distribution given by:

$$\mathscr{F}(\xi)=\frac{\xi_u\delta}{\xi(\xi_u-\xi)\sqrt{\pi}}\exp\left[-\left(\delta\ln\frac{\gamma\xi}{\xi_u-\xi}\right)^2\right]. \quad (24)$$

Equation (20) for the $NTU(z)$ written in terms of dimensionless quantities is:

$$NTU(\mathrm{H})=\frac{R}{R-1}\ln(1+(R-1)\bar{\theta}(\mathrm{H})) \quad (25)$$

where:

$$\bar{\theta}(\mathrm{H})=\int_o^{\xi_u}\theta(\xi,\mathrm{H})\mathscr{F}(\xi)\,d\xi. \quad (26)$$

Inspection of equations (25)–(26) demonstrates that NTU is a function of eight variables, H, R, ξ_u, γ, δ, υ, β and κ.

SOLUTION OF GOVERNING EQUATIONS

Equation (21) differs in two important ways from that arising in conventional counter flow tower analysis. First the square-bracketed part on the r.h.s. is not simply expressible in terms of $\theta(\xi,\eta)$ and hence the differential equation is not immediately separable, as is the case in conventional towers. Secondly, the curved-bracketed part of the r.h.s. is not constant with respect to η, as is usually assumed to be the case, but, through its dependence on $U(\xi,\eta)$, varies dramatically down the tower. In the solution of the equations which follows the effect of these two differences will be separated out: a simplified solution will first be arrived at in which the non-separable character of equation (21) will be essentially ignored, so that the contribution of the "non-constant curved bracket" part of the equation can be examined separately in detail. Then the solution to the full equations will be discussed, and the results presented as a correction factor on the simplified solution.

Simplified solution

Suppose it is assumed that due to some hypothetical mixing between droplets, all droplets at any given height have the same temperature so that $\theta(\xi,\eta)=\bar{\theta}(\eta)$ for all η. Then if one performs the operation $\int_o^{\xi}\mathscr{F}(\xi)[\ ]\,d\xi$ on equation (21), assuming $\theta(\xi,\eta)=\bar{\theta}(\eta)$ on the r.h.s., there results:

$$\frac{\partial\bar{\theta}(\eta)}{\partial\eta}=\beta\int_o^{\xi_u}\frac{Nu\left(U,\kappa\frac{U^2}{\xi}\right)\mathscr{F}(\xi)}{\xi(U-\upsilon\xi)}d\xi.\,[1+(R-1)\bar{\theta}]$$

which is immediately separable and integrates to give:

$$NTU_o(\mathrm{H})=\beta R\int_o^{\xi}\frac{\mathscr{F}(\xi)}{\xi}\left[\int_o^{\mathrm{H}}\frac{Nu\left(U,\kappa\frac{U^2}{\xi}\right)}{(U-\upsilon\xi)}d\eta\right]d\xi. \quad (27)$$

(The subscript "o" on NTU_o indicates that it results from the simplified solution.) By introducing $\tau(\xi,\eta)$, the local dimensionless residence time of the droplet in the tower, given by:

$$\tau(\xi,\eta)=\xi\int_o^{\eta}\frac{d\eta}{U-\upsilon\xi} \quad (28)$$

equation (27) can be written as:

$$NTU_o(\mathrm{H})=\beta R\int_o^{\xi}\frac{\mathscr{F}(\xi)}{\xi^2}\left[\int_o^{\tau(\xi,\mathrm{H})}Nu\left(U,\frac{\kappa U^2}{\xi}\right)d\tau\right]d\xi. \quad (29)$$

$\tau(\xi,\eta)$ and $U(\xi,\eta)$ must be determined from equation (22).

The full simplified solution will contain two additional simplifying assumptions. The Nusselt number is not a strong function of η so that the first of these additional assumptions is to assume it constant with respect to η for the purposes of the inner integration and to evaluate it at an average relative velocity of the droplet in the tower, given dimensionally by $\hat{V}_r=Z/T+V_a$ and dimensionlessly by:

$$\hat{U}(\xi,\mathrm{H})=\left(\frac{\mathrm{H}}{\tau(\xi,\mathrm{H})}+\upsilon\right)\xi \quad (30)$$

so that the resultant average Nusselt number Nu is given by:

$$\hat{Nu}(\xi,\mathrm{H})=Nu\left(\hat{U},\frac{\kappa\hat{U}^2}{\xi}\right). \quad (31)$$

With this simplification, the only information required from equation (22) is the residence time, $\tau(\xi,\mathrm{H})$. Equation (29) now becomes:

$$NTU_o(\mathrm{H})=\beta R\int_o^{\xi_u}\left(\frac{\mathscr{F}(\xi)}{\xi^2}\right).[\tau(\xi,\mathrm{H})\hat{Nu}(\xi,\mathrm{H})]\,d\xi \quad (32)$$

which may be written:

$$NTU_o(\mathrm{H})=\frac{\beta R}{\xi_{31}^2}\frac{\int_o^{\xi_u}\left(\frac{\mathscr{F}(\xi)}{\xi^2}\right)[\tau(\xi,\mathrm{H})\hat{Nu}(\xi,\mathrm{H})]\,d\xi}{\int_o^{\xi}\left(\frac{\mathscr{F}(\xi)}{\xi^2}\right)d\xi}. \quad (33)$$

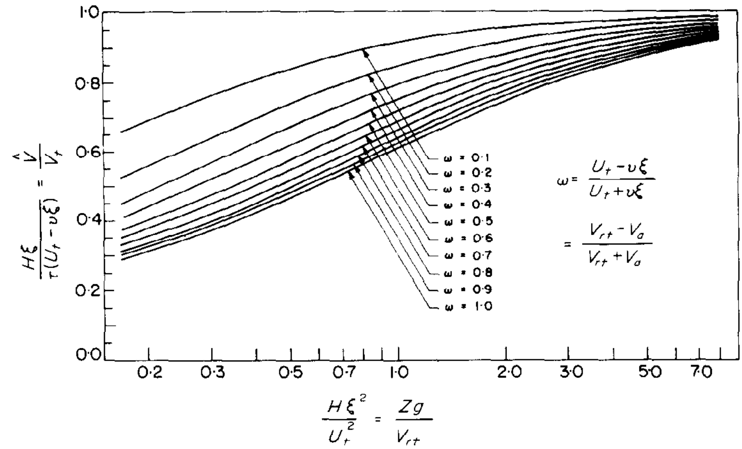

FIG. 2. Chart for estimating residence time of droplet in tower.

The second additional simplification is now made: it has been found that over the range of variables of practical interest, the product $\hat{N}u(\mathrm{H}, \xi) . \tau(\mathrm{H}, \xi)$ is relatively insensitive to ξ, the first part increasing, the second decreasing with ξ, and the product remaining relatively constant, typically to within 10 per cent over a five-fold range in ξ. The product will therefore be evaluated at the volume–diameter mean diameter, ξ_{31} so that we now have:

$$NTU_o(\mathrm{H}) = \frac{\beta R}{\xi_{31}^2} \tau(\xi_{31}, \mathrm{H}) \hat{N}u(\xi_{31}, \mathrm{H}). \quad (34)$$

This is the simplified solution to be used for comparison with the full solution.

The volume–diameter mean diameter ξ_{31} (or x_{31}) can be determined analytically for the upper limit droplet size distribution:

$$\xi_{31} = \xi_u \left[1 + 2\gamma \exp\left(\frac{1}{4\delta^2}\right) + \gamma^2 \exp\left(\frac{1}{\delta^2}\right) \right]^{-1}. \quad (35)$$

Written dimensionally equation (34) is:

$$NTU_o(Z) = 6 \left(\frac{m_L \rho_G}{m_G \rho_L}\right) \frac{\alpha \hat{N}u_{31}(Z)}{x_{31}{}^2} . T(x_{31}, Z). \quad (36)$$

The corresponding height of a transfer unit, $HTU_o = Z/NTU$ is given by:

$$HTU_o(Z) = \frac{1}{6} \left(\frac{m_G \rho_L}{m_L \rho_G}\right) . \frac{x_{31}^2}{\hat{N}u_{31}(Z)} \frac{1}{\alpha} . \hat{V}(x_{31}, Z) \quad (37)$$

where $\hat{V}(x_{31}, Z)$ is the average absolute velocity of the x_{31} droplet in the tower. In a packed tower, HTU is conventionally assumed to be independent of the tower height Z. In a spray tower this is clearly not the case since $\hat{V}_{31}$ varies dramatically with Z, particularly if as in the case of the present tower, the droplets start with zero absolute velocity.

In order to facilitate the evaluation of the residence time, Fig. 2 has been prepared. It gives the ratio of the absolute average velocity, $\hat{V}$, to the average terminal velocity V_t as a function of Z, g, V_a and V_{rt}. The plot is approximate since it is based upon integration of equation (22) but using a constant value of C_D,—that value for C_D which the droplet experiences at the relative terminal velocity. Since typically the drag on the droplet is considerably less than the gravitational force until the droplet is close to its terminal velocity, this technique is quite accurate and Fig. 2 has been found to be acceptable for engineering calculations. Further details of the plot are given in Appendix C. Figure 2 must be used in conjunction with a relation for the relative terminal velocity of a droplet as a function of the droplet size. This is obtained by setting the r.h.s. of equation (22) equal to zero, i.e. by setting

$$\xi^2 = \frac{3}{4} \frac{C_D\left(U, \kappa \frac{U^2}{\xi}\right) . U^2}{\xi} \quad (38)$$

and solving for U by trial and error. Figure 3 shows the result for $\kappa = 1{\cdot}250 \times 10^{-4}$, its value at 20°C. Alternatively one can use dimensional relations used by meteorologists for raindrops, in particular one given by Foote and DuToit [9] which is said to be accurate to within $\pm 0{\cdot}03$ m/s for drop diameters from 0·1 to 5·8 mm. These two methods have been found by the author to agree within $\pm 2{\cdot}5$ per cent. Foote and DuToit also gives a correction (small) for temperatures different from 20°C.

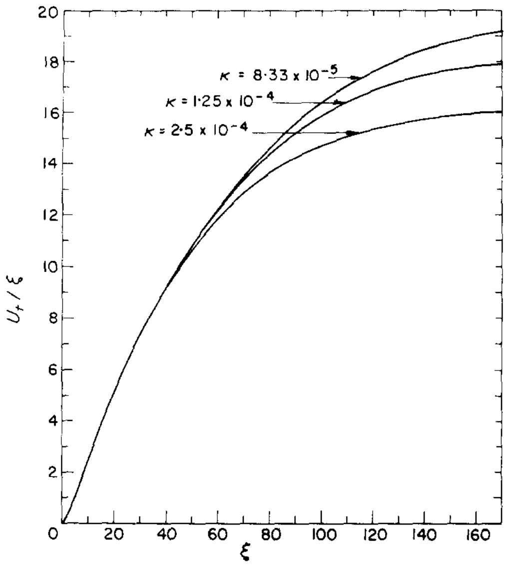

FIG. 3. Chart for determining terminal Reynolds number of droplet.

equations were then solved using a Runga–Kutta method, starting from the top of the tower and marching down. A Gauss–Siedel quadrature procedure [10] was used for the evaluation of the integral in equation (21). Because of the accuracy of this integration procedure it was found possible to obtain sufficient accuracy in the NTU by using six discrete regions for the droplet spectrum. The computer program was subjected to a number of internal checks for accuracy and consistency by comparing the results to that predicted with analytical procedures in the special cases where analytical results are possible. For example, for certain C_D relations (e.g. $C_D = A + B/Re$ where A and B are constants) equation (22) can be integrated analytically; the results compared favourably to that resulting from numerical integration using the same drag law. The fact that if the Nusselt number is assumed constant, C_D is assumed zero, and R is very large, then equation (34) is exact permitted another check on the numerical integration.

Figure 4 shows the results of the numerical solution for one (typical) set of the dimensionless variables in terms of the NTU and HTU. Because of the range of NTU covered in a graph such as Fig. 4 it is useful to present the results of the numerical solution of the full equations in terms of a correction factor on the NTU resulting from the simplified solution:

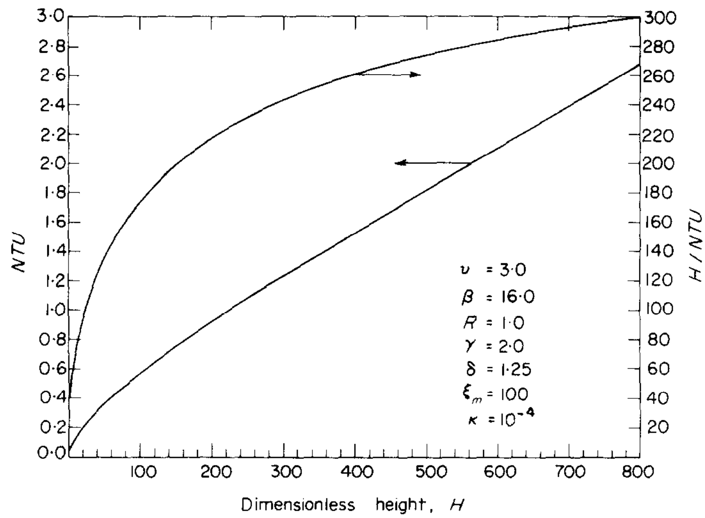

FIG. 4. Calculated NTU and the dimensionless HTU (= H/NTU) as a function of the tower height for one set of conditions.

Complete solution

The full set of governing equations ((21)–(24), (3) and (16)) have been solved numerically on a digital computer. The droplet size spectrum was approximated as a set of discrete regions, each having uniform diameter, and equations (21) and (22) written on each region. The resultant simultaneous ordinary differential

$$NTU = F_T NTU_o \tag{39}$$

where NTU_o is given by equation (34). Figures 5–9 show plots of F_T against NTU. They show the effect of varying each of the other parameters (except κ and β) over the practical range of interest, while maintaining the rest (except η) constant at central values. The effect

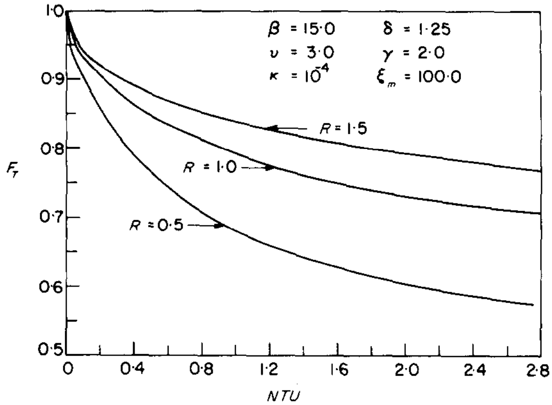

FIG. 5. Plot showing the effect of R on F_T.

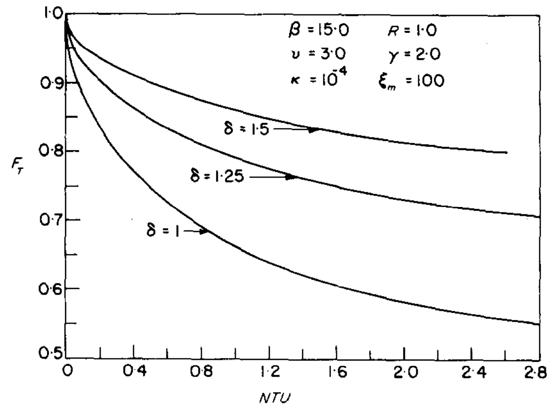

FIG. 6. Plot showing the effect of δ on F_T.

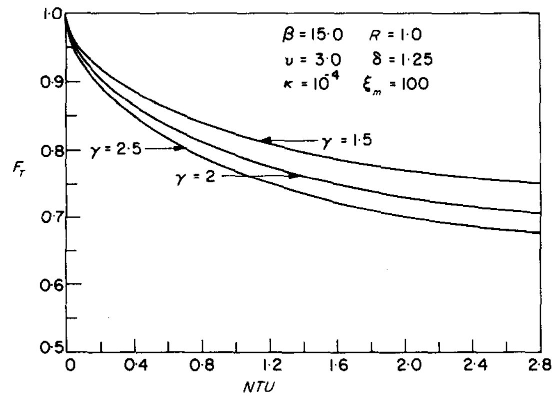

FIG. 7. Plot showing the effect of γ on F_T.

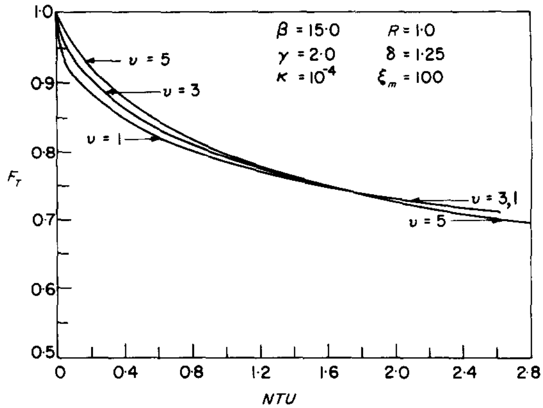

FIG. 8. Plot showing the effect of υ on F_T.

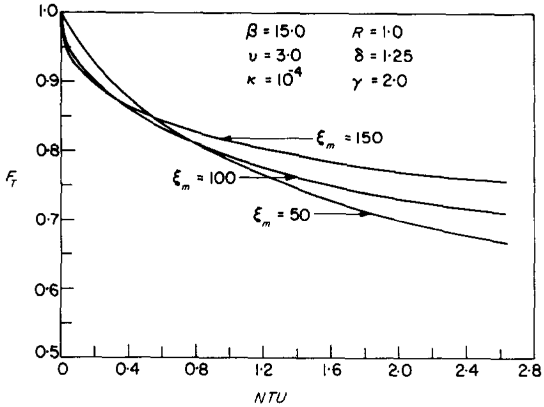

FIG. 9. Plot showing the effect of ξ_m on F_T.

on F_T of variation of κ and β over the range of practical interest ($0{\cdot}5 \times 10^{-4} < \kappa < 1{\cdot}5 \times 10^{-4}$; $5 < \beta < 30$) was found to be insignificant. (The parameter ξ_m is used in these figures rather than ξ_u to indicate the droplet size scale. ξ_m is the dimensionless volume–median diameter and is equal to $\xi_u/(1+\gamma)$.)

It will be noted that F_T is everywhere less than unity. This is because the simplified solution permits the "coefficient" (curve bracketed part of equation (21)) to vary with droplet size but not the "driving force"; (square bracketed part). In fact the smaller droplets, having a high surface to volume ratio and long residence times, suffer a greater temperature change in passing through the tower than the larger, for which the reverse is the case. Consequently the "driving force" for the smaller droplets is actually less than the average driving force, and that of the larger, greater than average. Since the predominate heat transfer in the simplified solution takes place from the smaller droplets, this solution overestimates the total heat transfer, and consequently represents an upper bound for the

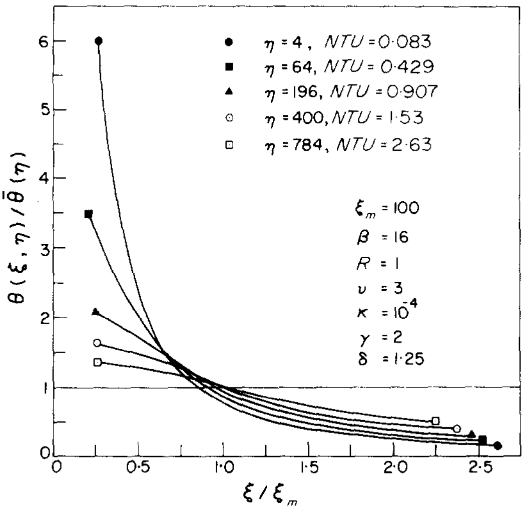

FIG. 10. Temperature distribution with respect to droplet size at various distances down the tower.

NTU. Figure 10 shows how the droplet temperature varies with height and droplet size for a given set of conditions. Another interesting feature of Figs. 5–9 is the fact that F_T appears to approach unity as *NTU* approaches zero. The reason for this can be seen from inspecting equation (21), for if *NTU* is small, $\theta(\xi, \eta)$ is small and hence θ can be neglected in the square bracketed part of this equation, making the two solutions essentially identical.

Inspection of Figs. 5–9 shows that F_T for a given *NTU* is relatively insensitive to υ, γ and ξ_m but strongly dependent on δ and R. This is to be expected in view of the above explanation since δ is a measure of the standard deviation of the droplet size distribution and R is the ratio of the slopes of the operating and equilibrium lines. If R is less than unity, the mean driving force decreases as one proceeds down the tower and therefore the effect of the driving force being different for different droplet sizes is more pronounced. For R greater the unity the reverse is the case. Figure 11 examines more closely the inter-effect of R and δ on F_T. Clearly very substantial errors can be introduced by assuming all droplets to be of the same size.

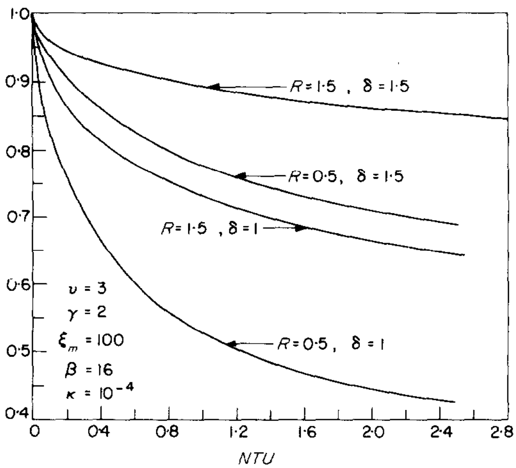

FIG. 11. Plot showing the inter-effect of R and δ on F_T.

EXPERIMENTAL STUDY

Experimental studies on spray cooling towers reported in the literature either depart from the type of tower treated here or do not report all relevant variables so that, unfortunately, direct comparison with the present theory is impossible. Testing the theory using experimental towers in the laboratory is troubled by the fact that, due to the necessarily restricted working cross-sectional area, uniform motion over a practical height is difficult to achieve and it is therefore difficult to avoid the migration to the wall of a good fraction of the droplets. For this reason testing of full-scale towers in the field is to be preferred although such tests are likely to lack the desired control of all variables.

A test has been carried out by the author on an on-line industrial tower and the results of this test will now be reported. The tower was of the type sketched in Fig. 1(b), so that the water droplets rose several feet from the spray before turning and falling down the tower. In applying the present theory in predicting the performance of the tower the heat transfer occurring while the droplets are rising will be neglected. This neglect will be compensated for to some extent by the fact that, although the tower had a relatively, large working cross-section (20·4 m^2) some droplets undoubtedly struck and flowed down the walls where they receive very little additional cooling.

The tower parameters were: $Z = 10{\cdot}5$ m; $m_L = 55{\cdot}5$ kg/s; $m_G = 71{\cdot}0$ kg/s; $V_a = 2{\cdot}9$ m/s; $\gamma = 2{\cdot}21$; $\delta = 1{\cdot}13$, $x_m = 2{\cdot}74$ mm. The last three variables were obtained from the spray nozzle manufacturer's data on the droplet size distribution for the nozzle used (Spraying Systems Ltd. nozzle No. $1\frac{1}{2}$C25 at 10 lb/in^2). An energy balance on measurements of inlet and outlet water temperature and air enthalpies gave an unaccounted energy rate of less than 3 per cent of the total duty of the tower. The measured *NTU* was 0·635 (average of two independent tests which gave values of 0·626 and 0·643 respectively). The separation between the equilibrium and operating lines was large in this case so that equation (20) was applicable. Hence the measured *NTU* was determined from this equation using measured values of H_{ao}, H_{so}, $\bar{H}_s(Z)$ and $H_a(Z)$. The water cooling range was from approximately 35 to 25C.

The dimensionless parameters for this tower are: $R = 0.556$; $\beta = 11.7$; $H = 351$; $\upsilon = 5.4$; $\gamma = 2.21$; $\delta = 1.13$, $\xi_m = 96.9$. According to the present analysis the predicted NTU for these parameters is 0·684, which is within 7·5 per cent of the measured value. The corresponding value for $F_T = 0.69$. In view of the neglect of the "rise region" and of the droplet deposition on the walls and in view of the uncertainties in the values of x_m, γ and δ, this agreement should not be considered a verification of the theory. However, it does constitute an indication that the theory is at least capable of estimating the performance of real towers.

CONCLUSIONS

1. It is clear that at least two assumptions made in the conventional analysis of cooling towers are not applicable to spray cooling towers—namely uniformity of liquid temperature at a given height, and invariance of the surface area per unit volume, and surface coefficients with respect to distance down the tower. (Whether these assumptions apply to a more conventional packed tower is difficult to say at the present time due to lack of measurements on droplet size distribution in these towers.) The deviations from the conventional analysis are so large that it appears unwise to couch experimental results in terms of the HTU or overall enthalpy volumetric coefficients.

2. The analysis presented here should be sufficiently accurate for the design of spray towers of the type sketched in Fig. 1. However, it should be noted that in practice considerable care is required in the layout of the nozzles to ensure as little as possible of the spray migrates to the walls.

3. Due to the predicted decrease in performance resulting from a wide droplet-size distribution, an atomizer producing as uniform a droplet size as possible should be used.

4. It is clear from equation (36) that as small a mean droplet size as possible is desirable for high performance. However, in practice, as was pointed out in the Introduction, as the mean diameter is decreased, the fraction of small droplets whose relative terminal velocity is less than the air velocity and hence which travel upward and are removed by the eliminators, becomes larger as the mean droplet size is reduced. Thus, dealing with the typical droplet size distributions produced by available nozzles suitable for cooling towers, and allowing say 1 per cent of the total water flow to be carried up to the eliminators, sprays with mean droplet size of the order of 1–2 mm must be used. An atomizer producing a spray of uniform droplets would not encounter the problem and hence a smaller mean drop diameter could be used. This again augers well for using an atomizer producing uniform droplet size.

Acknowledgements—This study was supported in part by a grant from the National Research Council of Canada. Part of this study was carried out while the author was under the employment of the Commonwealth Scientific and Industrial Research Organization, (Division of Mechanical Engineering) Melbourne, Australia. The author is indebted to Mr. R. A. Wallis of the above establishment for pointing out the industrial importance of the problem, Mr. R. V. Dunkle for many helpful discussions and Mr. M. Kaphammel for assistance in the computer programming.

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APPENDIX A

Evaluation of $C_D(Re, We)$

Water droplets of the order of size of interest in this study (~ 1 mm) distort from the spherical shape near their terminal velocities and Hughes and Gilliland [6] fitted a wide range of experimental data covering a range of liquids by use of a method based on assuming that the distorted shape is an oblate spheroid. The slenderness ratio λ of the spheroid was shown to be expressible as a function of the group $Re^{0.35}.We$:

$$\lambda = \lambda(Re^{0.35}.We) \tag{A-1}$$

and they tabulate this function. The details of the actual determination for a given We and Re may be found in the reference: suffice it to say that for the drag on spheres, the expression due to Marshall [11]:

$$C_D^s = 0.22 + \frac{24}{Re}(1+0.15\,Re^{0.6}) \tag{A-2}$$

was used in the present study and for the drag on a disk the expression

$$C_D^D = 1.12+(20/Re)+0.66/(1+17.5(\log_{10}(Re/280))^2) \tag{A-3}$$

was used and that a C_D based upon a weighted average of these two, due to Riabouchinsky [12] gave the drag coefficient on the droplet. This drag law has been found to be consistent with data obtained by meterologists, as summarized by Foote and DuToit [9] for the terminal velocity of raindrops.

APPENDIX B

Evaluation of Nu(Re, We)

The Reynolds number range of interest for droplets falling from rest to their terminal velocity is approximately 300–2000. Experiments measuring convective coefficients over spheres governing this range have been extensively reviewed by Rowe *et al.* [13]. Due to its simplicity, and due to its correctness in the limit, $Re \to 0$, the correlation equation

$$Nu = 2+BPr^{1/3}Re^{1/2} \tag{B-1}$$

is preferred although it is now realized that the exponents on both the Reynolds number and the Prandtl number both depend on "Re" and "Pr". For studies using air as the fluid (in both heat and mass transfer) values of "B" of 0·552, 0·555, 0·547, 0·600 and 0·690 have been obtained, as reported in references [14–17 and 13] respectively. The value of 0·6 due to Ranz and Marshall [7] was chosen for the present study. As has been mentioned, large droplets experience some distortion from the spherical shape. Effect of this on the heat transfer has been investigated by assuming, following Hughes and Gilliland [6], that the droplet is an oblate spheroid. The mean convective coefficient over an oblate spheroid has been investigated by Skelland and Cornish [17] who found that their data could be fitted to that of a sphere provided one used an equivalent diameter for the oblate spheroid equal to its total surface area divided by its perimeter normal to the flow, denoted by x_1. The value of x_1 can be shown by geometric arguments to be given by:

$$x_1 = \frac{x}{2\lambda^{1/3}}\left[1+\frac{\lambda^2}{\sqrt{(1-\lambda^2)}}\right]\ln\left[\frac{1+\lambda+\sqrt{(1-\lambda^2)}}{1+\lambda-\sqrt{(1-\lambda^2)}}\right] \tag{B-2}$$

where λ is the slenderness ratio obtained from equation (A-1). The Nusselt number for the droplet can then be shown to be given by:

$$Nu(Re, We) = \frac{1}{\lambda^{1/3}}\left[2+0.6\,Pr^{1/3}Re^{1/2}\left(\frac{x_1}{x}\right)^{1/2}\right]. \tag{B-3}$$

(A value of 0·72 was used for the Prandtl number.)

Equations (A-1), (B-2) and (B-3) were used for determining the Nusselt number. In fact, the corrections to the Nusselt number associated with the drop's non-spherical shape outlined above was found to be quite small (less than 5 per cent).

APPENDIX C

Residence Time for Constant C_D

From equation (38) the value of C_D at the terminal velocity is:

$$C_D = \frac{4}{3}\frac{\xi^3}{U_t}. \tag{C-1}$$

Substitution of this value for C_D into equation (22) gives:

$$\frac{\partial U}{\partial \eta} = \frac{\xi^2}{U-\upsilon\xi}\left(1-\left(\frac{U}{U_t}\right)^2\right) \tag{C-2}$$

and integration of the differential equation from $\eta = o$ (where $U = v\xi$) to $\eta = x$ gives:

$$\frac{\xi^2 \mathrm{H}}{U_t^2} = \int_o^\psi \frac{x\,dx}{(1-x)\left(x+\frac{1}{\omega}\right)} \tag{C-3}$$

where:

$$\psi = \frac{U-\upsilon\xi}{U_t-\upsilon\xi}. \tag{C-4}$$

Substituting equation (28) for η into equation (C-2) and integrating gives:

$$\frac{\xi(U_t-v\xi)\tau(\mathrm{H},\xi)}{U_t^2} = \int_o^\psi \frac{dx}{(1-x)\left(x+\frac{1}{\omega}\right)}. \tag{C-5}$$

Equations (C-3) and (C-5) represent an implicit relation between H and τ, relating them through the parameter ψ. This relation is graphed in Fig. 2.

ANALYSE D'UNE TOUR DE REFROIDISSEMENT A CONTRE-COURANT

Résumé—La tour de refroidissement étudiée est sans garnissage. Elle est verticale, le courant d'air s'élevant uniformément tandis que le courant d'eau dispersé en gouttelettes descend uniformément. Les gouttes sont introduites au sommet de la tour à vitesse nulle, à température uniforme, la distribution en taille étant connue. On tient compte du fait que les gouttes à une même hauteur n'ont pas la même température. Les résultats sont présentés sous la forme d'un facteur de correction qui affecte la solution simple obtenue en négligeant ce fait. On examine l'effet du facteur de correction de tous les groupes adimensionnels caractéristiques.

DIE UNTERSUCHUNG EINES TROPFEN-KÜHLTURMES IN GEGENSTROMAUSFÜHRUNG

Zusammenfassung—Der hier untersuchte Kühlturm hatte keine Einbauten. Der Kühlturm arbeitet mit senkrechter Stromführung, und zwar mit gleichmäßig verteilter, aufwärts strömender Luft sowie einem gleichmäßig verteilten, abwärts gerichteten Wasserstrom, der in Tropfen zerstäubt ist. Die Tropfen haben am Kühlturmeintritt die Anfangsgeschwindigkeit Null, gleiche Temperatur und ein bekanntes Tropfenspektrum. Die Untersuchung berücksichtigt, daß bei den verschiedenen Fallhöhen die Tropfen nicht die gleiche Temperatur besitzen. Die Ergebnisse werden mit einer vereinfachten Gleichung dargestellt, die den Temperaturunterschied vernachlässigt und daher mit einem Korrekturfaktor erweitert ist. Die Auswirkungen aller in Frage kommenden dimensionslosen Kenngrößen auf den Wert des Korrekturfaktors werden überprüft.

АНАЛИЗ ПРОТИВОТОЧНОЙ БРЫЗГАЛЬНОЙ ГРАДИРНИ

Аннотация — Анализируется полая вертикальная градирня с потоком воздуха, равномерно движущимся вверх, и потоком воды в виде капель, равномерно движущегося вниз. Капли поступают в верхнюю часть градирни с нулевой скоростью, однородной температурой и известным распределением по размерам. При анализе принимается во внимание, что температура капель на разных высотах неодинакова. Результаты представлены в виде поправочного коэффициента к упрощенному решению, в котором не учитывается этот факт. Исследуется влияние соответствующих безразмерных групп на поправочный коэффициент.